MATRICES

Introduction :-

A large number of physical phenomenan are stated by a linear differential equations which are solved by Reducing them into a system of simultaneous linear equations

At present we have found a wide Applications in Matrices. They play a vital role not only in Mathematics but also in Computer graphics, optics, Chryptography and Encryption, Robotics and animations etc. particularly Matrices play the key roll in providing suitable creteria for testing the consistancy of a system of linear equations.

Matrix :- A set of mn numbers (Real or complex) arrange in a rectangular formation having m rows and n columns and bounded by the brackets ( ), [ ], { }. is called an m/n matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{i1} & a_{i2} & a_{i3} & \cdot\ a_{ij} \cdots & a_{in} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

The above matrix can be written as

$A = [a_{ij}]_{m \times n}$ $\quad i = 1, 2, 3, \ldots m$
$\quad\quad j = 1, 2, 3, \ldots n$

Singular matrix :

• A matrix "A" is said to be singular than $\det|A| = 0$

If $|A| \neq 0$ They "A" is called Non-singular matrix.

Rank of a matrix

Rank of a matrix is the highest order of the minor of the matrix which is not zero

The Rank can be defined for both square and rectangular matrices.

properties:- ① The rank of the matrix is always unique

② The rank of null matrix is zero

③ The rank of the Identity matrix is its order of the matrix.

④ The rank of a non-singular matrix of order n is "n".

⑤ The rank of the singular matrix of order n is less Than "n"

Echelon form

A matrix is said to be in Echelon form if it is satisfies the following properties

① The first non-zero Element in any non-zero row of the matrix must be equal to unity.

② The upper triangular form of the matrix obtained by the Elementary row ~~and~~ column transformations on the matrix is known as the Echelon form

③ The Rank of a matrix is nothing but the number of non-zero rows in the Echelon form

Find the rank of the matrix

$$\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$

$R_3 \to R_3 + R_1$

$R_4 \to R_4 + 3R_1$

$$\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 5 & 5 & 0 & 5 \\ 0 & 10 & 10 & 0 & 10 \\ 0 & 15 & 15 & 0 & 15 \end{bmatrix}$$

$R_2/5$, $R_3/10$, $R_4/15$

$$\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$, $R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

rank = 2

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_1 \to R_2$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$\Rightarrow$ $R_2 \to R_2 - 2R_1$, $R_3 - 3R_1$, $R_4 \to R_4 - 6R_1$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_4 \to R_4 - (R_2 + R_3)$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to 5R_3 - 4R_2$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank (A) $\rho = 3$

Find the rank of the matrix

$$A = \begin{bmatrix} 10 & -2 & 3 & 0 \\ 2 & 10 & 2 & 4 \\ -1 & -2 & 10 & 1 \\ 2 & 3 & 4 & 9 \end{bmatrix}$$

$\Rightarrow$ $R_1 \to R_1/2$ $\begin{bmatrix} 10 & -2 & 3 & 0 \\ 2 & 10 & 2 & 4 \\ -1 & -2 & 10 & 1 \\ 2 & 3 & 4 & 9 \end{bmatrix}$

$R_1 \to R_2/2$

$$\begin{bmatrix} 1 & 5 & 1 & 2 \\ 10 & -2 & 3 & 0 \\ -1 & -2 & 10 & 1 \\ 2 & 3 & 4 & 9 \end{bmatrix}$$

$R_2 \to R_2 - 10R_1$

$R_3 \to R_3 + R_1$

$R_4 \to R_4 - 2R_1$

$$\begin{bmatrix} 1 & 5 & 1 & 2 \\ 0 & -52 & -7 & -20 \\ 0 & 3 & 11 & 3 \\ 0 & -7 & 2 & 5 \end{bmatrix}$$

$R_4 \to 3R_4 + 7R_3$

$$\begin{bmatrix} 1 & 5 & 1 & 2 \\ 0 & -52 & -7 & -20 \\ 0 & 0 & 11 & 3 \\ 0 & 0 & 83 & 36 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Normal form (or) Cannonical form

A matrix of the form $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ where I_r is a unit matrix of order "r" and "0" is the null matrix is called the Normal form (or) First cannonical form

The matrices of the form

$[I_r \ 0]$, $\begin{bmatrix} I_r \end{bmatrix}$, $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$ are also known as the normal form.

→ At every $m \times n$ matrix can be reduced to the normal form by a series of Elementary row and column transformations both.

→ The rank of the above matrix is "r".

Reduce the matrix $\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$ into canonical form and find rank.

$$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$$

$R_3 \to R_3 + R_1$

$$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

$C_1/8, \; R_3/10$

$$\begin{bmatrix} 1 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_2 \to C_2 - C_1, \; C_3 \to C_3 - 3C_1$

$C_4 \to C_4 - 6C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_2/3, \; C_3/2, \; C_4 \to C_4 - 2C_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to C_3 - C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_3 \to C_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} I_3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1$

$C_3 \to C_3 - 3C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 / -4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & 1 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & 1 & -5 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 - C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 / 2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to 3C_4 + C_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_3 \to C_3 + 5C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = I_3 = 0$$

Reduce the mattix into caunical form

$$\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

$\Rightarrow R_3 \to R_1$

$$\begin{bmatrix} 1 & -1 & 0 & 3 \\ 4 & 2 & 0 & 2 \\ 2 & -2 & 0 & 6 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

$\Rightarrow R_1/2,\ R_2/2$

$$\begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 1 & 0 & 1 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

$\Rightarrow R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 3 & 0 & 5 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

$\Rightarrow R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 3 & 0 & 5 \\ 0 & -1 & 0 & 3 \\ 0 & -1 & 1 & -1 \end{bmatrix}$$

$\Rightarrow R_1/2,\ R_2/2$

$$\rightarrow \begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & 1 & 0 & 1 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

$\Rightarrow R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - R_1$

$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 3 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix}$$

$C_2 \to C_2 + C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 3 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix}$$

Linear system of Equations

The general system on non-homogeneous equations in the matrix nation can be written as $AX = B$

Where $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$, $B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}$

The matrix equation $AX = B$ need not always have a solution. It may have no solution, or a unique solution or many solutions.

Consistancy

→ The system of equations having one or more solutions is called a consistant system of equations.

→ A system of equations having no solution is called an Inconsistant system of Equations.

Conditions for consistancy

→ let the rank of a matrix be r, $\rho(A) = r$ and the rank of the Argumented matrix $AB = r'$ and Let the number of unknowns be "n"

① If $r \neq r'$, Then the system is inconsistant, Then system has no solution.

② If $r = r'$, then the system is consistant,

③ If $r = r' = n$, The the system is consistant and have a unique solution.

④ If $r = r' < n$ The the system is consistant and has an infinite number of solutions.

In this case $(n-r)$ ~~arbitrary~~ variables are considered arbitrary and remaing variables shall be uniquely determined.

7. test for consistancy for the following equations

$x+y+z=4$
$2x+5y-2z=3$
$x+7y-7z=5$ if so solve Them.

solution. The given equations can be written the matrix form

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & -2 \\ 1 & 7 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix} \Rightarrow AX=B \longrightarrow ①$$

now considered the argumend matrix AB

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 4 \\ 2 & 5 & -2 & 3 \\ 1 & 7 & -7 & 5 \end{bmatrix}$$

$R_2 - R_2 - 2R_1$
$R_3 - R_3 - R_1$

$$= \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 3 & -4 & -5 \\ 0 & 6 & -8 & 1 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_2$

$$= \begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 3 & -4 & -5 \\ 0 & 0 & 0 & 11 \end{bmatrix}$$

$\rho(A) = 2$, $\rho(AB) = 3$

$\rho(A) \neq \rho(AB)$

$r \neq r'$

therefore it is inconsistancy.

2. Solve the system of linear equations

$$x+y+2z=4$$
$$2x-y+3z=9$$
$$3x-y-z=2$$

Ans. The given equations can be written in the matrix form

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 3 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 2 \end{bmatrix} \Rightarrow AX=B \rightarrow (1)$$

Now Considered the argumend matrix AB

$$[AB] = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 2 & -1 & 3 & 9 \\ 3 & -1 & -1 & 2 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 3R_1$

$$= \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & -3 & -1 & 1 \\ 0 & -4 & -7 & -10 \end{bmatrix}$$

$R_3 \rightarrow 3R_3 - 4R_2$

$$= \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & -3 & -1 & 1 \\ 0 & 0 & -17 & -34 \end{bmatrix}$$

$R_3/17$

$$= \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & -3 & -1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \rightarrow (2)$$

$$e(A) = e(AB) = 3 = n\ (3)$$

The system is consistaule and has a unique solution.

Equation ② can be written as

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -3 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$\Rightarrow$ $x+y+2z = 4$

$-3y - z = 1$

$\boxed{z = 2}$

$\Rightarrow$ $-3y - 2 = 1$

$-3y = 3$

$\boxed{y = -1}$

$\Rightarrow$ $x - 1 + 4 = 4$

$\Rightarrow$ $x - 1 = 0$

$\boxed{x = 1}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

3. Test for consistency for the following equations

$5x + 3y + 7z = 4$

$3x + 26y + 2z = 9$

$7x + 2y + 10z = 5$ If so solve them.

Ans. The equations can be written in the form of matrix form

i.e, $$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix} \Rightarrow AX = B \rightarrow ①$$

$\Rightarrow$ now considered the argument matrix of AB

$$\begin{bmatrix} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{bmatrix}$$

$R_2 \to 5R_2 - 3R_1$

$R_3 \to 5R_3 - 7R_1$

$$\begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{bmatrix}$$

$R_2/11 \Rightarrow \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & -11 & 1 & -3 \end{bmatrix}$

$R_3 \to R_3 + R_2 \Rightarrow \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \to (2)$

$\rho(A) = \rho(AB) = 2 < n(3)$

The system is consistant but has a infinite number of solutions

$n - r = 3 - 2 = 1$ Linearly Independent solution

arbitrary constant

Equation (2) can be written as

$$\begin{bmatrix} 5 & 3 & 7 \\ 0 & 11 & -1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$$

$5x + 3y + 7z = 4$

$11y - z = 3$

$z = 0$

$\Rightarrow 11y - z = 3$

$\Rightarrow 11y = 3$

$y = 3/11$

let $z = k$

$y = \frac{3+k}{11}$, $x = \frac{7-16k}{11}$

$5x + 3\left(\frac{3+k}{11}\right) + 7k = 4$

$\Rightarrow 5\left(\frac{7-16k}{11}\right) + 3\left(\frac{3+k}{11}\right) + 7k = 4$

$\Rightarrow \frac{35 - 80k + 9 + 3k + 77k}{11} = 4$

$\Rightarrow$

1. Discuss for what values of λ and μ for the given Equations

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

have

(1) no solution

(2) unique solution

(3) an infinite no. of solutions

Sol. The equations can be written in the matrix notation

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix} \Rightarrow AX=B \rightarrow (1)$$

Now Argumented matrix

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{bmatrix}$$

$\Rightarrow R_3 \rightarrow R_2$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$$

(1) no solution.

The rank of $A \neq$ The rank of AB

$\Rightarrow \rho(A) \neq \rho(AB)$

$\Rightarrow \lambda-3=0 \qquad \mu-10 \neq 0$

$\Rightarrow \lambda=3 \qquad \mu \neq 10$

(ii) unique solution

$$\rho(A) = \rho(AB) = n(3)$$

$\lambda - 3 \neq 0$ for any value of μ

$\lambda \neq 3$

Then the system has unique solution.

(iii) Infinitely many solutions

$$\rho(A) = \rho(AB) < n(3)$$

$\lambda - 3 = 0$, $\mu - 10 = 0$

$\lambda = 3$, $\mu = 10$

2. Invistigiate for what values of a, b

$$x + 2y + 3z = 4$$
$$x + 3y + 4z = 5$$
$$x + 3y + az = b$$

Ans ⇒ The equation system of equations can be written in the matrix notation

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 3 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ b \end{bmatrix} \Rightarrow AX = B \rightarrow ①$$

Now argumented matrix

$$[AB] = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 5 \\ 1 & 3 & a & b \end{bmatrix}$$

$\Rightarrow R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & a-3 & b-4 \end{bmatrix}$$

$\Rightarrow R_3 \rightarrow R_3 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & a-4 & b-5 \end{bmatrix}$$

(i) system has Infinitely many solution

$\rho(A) = \rho(AB) < n$ (3)

$\Rightarrow a-4 = 0, \quad b-5 = 0$

$a = 4, \quad b = 5$

(ii) For no solution

$\rho(A) \neq \rho(AB)$

$a-4$

5. Find the value of λ for which the system of equations $3x - y + 4z = 3$, $x + 2y - 3z = -2$, $6x + 5y + \lambda z = -3$ will have an infinite number of solutions. and solve them with the value of λ.

Ans. The system of equations can be written in the matrix form

$$\begin{bmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix} \Rightarrow AX = B \rightarrow (1)$$

Now

$$[AB] = \begin{bmatrix} 3 & -1 & 4 & 3 \\ 1 & 2 & -3 & -2 \\ 6 & 5 & \lambda & -3 \end{bmatrix}$$

$\Rightarrow$ $R_2 \rightarrow 3R_2 - R_1$

$R_3 \rightarrow R_3 - 2R_1$

$\Rightarrow R_1 \to R_2$

$$\begin{bmatrix} 1 & 2 & -3 & -2 \\ 3 & -1 & 4 & 3 \\ 6 & 5 & \lambda & -3 \end{bmatrix}$$

$\Rightarrow$ $R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 6R_1$

$$[AB] = \begin{bmatrix} 1 & 2 & -3 & -2 \\ 0 & -7 & 13 & 9 \\ 0 & 0 & \lambda+5 & 0 \end{bmatrix} \to (2)$$

given that The system has an infinite number of solutions

$\Rightarrow \rho(A) = \rho(AB) < n(B)$

$\Rightarrow \lambda+5=0$

$\Rightarrow \lambda=-5$

Now

The equation (2) can be written as

$$\begin{bmatrix} 1 & 2 & -3 \\ 0 & -7 & 13 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \\ 0 \end{bmatrix}$$

$\Rightarrow x+2y-3z=-2$

$-7y+13z=9$

$\Rightarrow n-r = 3-2=1$ Linearly Independent

let $z=k$

$-7y+13k=9$

$\Rightarrow y = \frac{9-13k}{-7}$

$\Rightarrow$

Gauss Elimination method

Solve the system of equations by using Gauss Elimination method

$3x+y+2z=3$
$2x-3y-z=-3$
$x+2y+z=4$

Ans. The given system of equations are be written in the form of matrix notation

$$\Rightarrow \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix} \Rightarrow AX=B \longrightarrow (1)$$

$\Rightarrow$ Now

$$[AB] = \begin{bmatrix} 3 & 1 & 2 & 3 \\ 2 & -3 & -1 & -3 \\ 1 & 2 & 1 & 4 \end{bmatrix}$$

$R_3 \leftrightarrow R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & -3 & -1 & -3 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$

$\Rightarrow R_2 \longrightarrow R_2-2R_1$

$R_3 \longrightarrow R_3-3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & -5 & -1 & -9 \end{bmatrix}$$

$\Rightarrow R_3 \leftrightarrow 7R_3 - 5R_2$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & 0 & 8 & -8 \end{bmatrix}$$

$\Rightarrow R_3/8$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & 0 & 1 & -1 \end{bmatrix} \rightarrow (2)$$

$\rho(A) = 3 = \rho(AB) = 3$

$$\Rightarrow \begin{aligned} x + 2y + z &= 4 \\ -7y - 3z &= -11 \\ z &= -1 \end{aligned}$$

$\Rightarrow z = -1 \Rightarrow -7y + 3 = -11$

$-7y = -11 - 3$

$\therefore 7y = -14$

$y = 2$

$x + 4 - 1 = 4$

$x + 3 = 4$

$x = 1$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$\begin{aligned} x_1 + x_2 + 2x_3 + x_4 &= 6 \\ 6x_1 - 6x_2 + 6x_3 + 2x_4 &= 36 \\ 4x_1 + 3x_2 + 3x_3 - 3x_4 &= -1 \\ 2x_1 + 2x_2 - x_3 + x_4 &= 10 \end{aligned}$$

$$\Rightarrow [AB] = \begin{bmatrix} 1 & 1 & 2 & 1 & 6 \\ 6 & -6 & 6 & 2 & 36 \\ 4 & 3 & 3 & -3 & -1 \\ 2 & 2 & -1 & 1 & 10 \end{bmatrix}$$

$\Rightarrow R_2 \to R_2 - 6R_1$

$R_3 \to R_3 - 4R_1$

$R_4 \to R_4 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 & 1 & 6 \\ 0 & -12 & -6 & -4 & 0 \\ 0 & -1 & -3 & -7 & -25 \\ 0 & 0 & -5 & -1 & -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 & 1 & 6 \\ 0 & -12 & -6 & -4 & 0 \\ 0 & -1 & -5 & -7 & -25 \\ 0 & 0 & -5 & -1 & -2 \end{bmatrix}$$

$\Rightarrow R_3 \rightarrow R_3 + R_1$

$$= \begin{bmatrix} 1 & 1 & 2 & 1 & 6 \\ 0 & -12 & -6 & -4 & 0 \\ 0 & 0 & -1 & -6 & -19 \\ 0 & 0 & -5 & -1 & -2 \end{bmatrix}$$

$R_4 \rightarrow R_4 - 5R_3$

$R_2 / -2$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 2 & 1 & 6 \\ 0 & 6 & 3 & 2 & 0 \\ 0 & 0 & -1 & -6 & -19 \\ 0 & 0 & 0 & 29 & 93 \end{bmatrix}$$

$$\Rightarrow [AB] = \begin{bmatrix} 1 & 1 & 2 & 1 & 6 \\ 0 & -\cancel{12} & -6 & -4 & 0 \\ 0 & -1 & -3 & -7 & -25 \\ 0 & 0 & -5 & -1 & -2 \end{bmatrix}$$

$$R_3 \rightarrow R_3 + R_1 = \begin{bmatrix} 1 & 1 & 2 & 1 & 6 \\ 0 & -2 & -6 & -4 & 0 \\ 0 & 0 & -1 & -6 & -19 \\ 0 & 0 & -5 & -1 & -2 \end{bmatrix}$$

$R_4 \rightarrow 2R_4 + 5R_1$

$$= \begin{bmatrix} 1 & 1 & 2 & 1 & 6 \\ 0 & -2 & -6 & -4 & 0 \\ 0 & 0 & -1 & -6 & -19 \\ 0 & 0 & 0 & 3 & 26 \end{bmatrix}$$

$R_4 \rightarrow R_4 + R_3$

$$= \begin{bmatrix} 1 & 1 & 2 & 1 & 6 \\ 0 & -2 & -6 & -4 & 0 \\ 0 & 0 & -1 & -6 & -19 \\ 0 & 0 & 0 & -3 & 7 \end{bmatrix}$$

Gauss-Jordan Method

using Gauss-Jordan Method solve the system of Equations

$$2x+y+z=10$$
$$3x+2y+3z=18$$
$$x+4y+9z=16$$

The given equations can be written in the matrix notation

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix} \Rightarrow AX=B$$

$$\boxed{AB} = \begin{bmatrix} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{bmatrix}$$

$R_1 \longleftrightarrow R_3$

$$\begin{bmatrix} 1 & 4 & 9 & 16 \\ 3 & 2 & 3 & 18 \\ 2 & 1 & 1 & 10 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 3R_1$

$R_3 \rightarrow R_3 - 2R_1$

$$= \begin{bmatrix} 1 & 4 & 9 & 16 \\ 0 & -10 & -24 & -30 \\ 0 & -7 & -17 & -22 \end{bmatrix}$$

$R_2/-2$ $$= \begin{bmatrix} 1 & 4 & 9 & 16 \\ 0 & 5 & 12 & 15 \\ 0 & -7 & -17 & -22 \end{bmatrix}$$

$R_3/-1$ $$= \begin{bmatrix} 1 & 4 & 9 & 16 \\ 0 & 5 & 12 & 15 \\ 0 & 7 & 17 & 22 \end{bmatrix}$$

$R_3 \to 5R_3 - 7R_2$

$$\Rightarrow \begin{bmatrix} 1 & 4 & 9 & 16 \\ 0 & 5 & 12 & 15 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$R_2 \to R_2 - 12R_3$

$$\Rightarrow \begin{bmatrix} 1 & 4 & 9 & 16 \\ 0 & 5 & 0 & -45 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$\Rightarrow R_3/5$

$$= \begin{bmatrix} 1 & 4 & 9 & 16 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$\Rightarrow R_1 \to R_1 - 4R_2$

$$\begin{bmatrix} 1 & 0 & 9 & 52 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$\Rightarrow R_1 \to R_1 - 9R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -9 \\ 5 \end{bmatrix}$$

Solve the system of equations by using Gauss - Jordan method.

$10x + y + z = 12$
$2x + 10y + z = 13$
$x + y + 5z = 7.$

The given equations can be written in the form of matrix notation

$$\Rightarrow \begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \\ 7 \end{bmatrix} \Rightarrow AX = B \longrightarrow (1)$$

$\Rightarrow$ Now

$$[AB] = \begin{bmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{bmatrix} \Rightarrow$$

$R_1 \longleftrightarrow R_3$

$$= \begin{bmatrix} 1 & 1 & 5 & 7 \\ 2 & 10 & 1 & 13 \\ 10 & 1 & 1 & 12 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 10R_1$

$$= \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -48 \end{bmatrix}$$

$R_3 \longrightarrow 8R_3 - 9R_2$

$$= \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & -473 & -473 \end{bmatrix}$$

$R_3 / -473$

$$= \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$R_3 \times 9 = \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 9 & 9 \end{bmatrix}$$

$$[AB] = \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 9 & 9 \end{bmatrix}$$

$R_2 \longrightarrow R_2 + R_3$

$$= \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 8 & 0 & 8 \\ 0 & 0 & 9 & 9 \end{bmatrix}$$

$$R_2/8 \,|\, R_3/9 = \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 8 & 0 & 8 \\ 0 & 0 & 9 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

$$R_1 \to R_1 - R_2 = \begin{bmatrix} 1 & 0 & 5 & 6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$\Rightarrow \quad R_1 \longrightarrow R_1 - 5R_3.$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

HOMOGENEOUS LINEAR EQUATIONS

consider a system of equations

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = 0$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = 0$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n = 0$$

This is called a system of m homogeneouse Equations and "n" unknowns $x_1, x_2, \ldots x_n$

The above set of equations can be written in the matrix form.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow AX = 0$$

In the Homogenouse system it is always consistant

$$\Rightarrow r = r'$$

Here it may be noted that sometimes $x = 0, \Rightarrow x_1 = x_2 = \cdots x_n$

it is called a trivial solution.

TYPES OF SOLUTIONS

① The system is always consistant, so it has solution.

② If $r = n$, Then the system is consistant and has a trivial solution and ~~so~~ is unique.

(ii) If $r < n$ Then the System has a non-trivial Solution and has an infinite number of solutions.

NOTE:

A homogeneous system $AX = 0$ posses a non-trivial solution. If and only if $|A| = 0$

problem

1) Solve The system of equations.

$$2w + 3x - y - z = 0$$
$$4w - 6x - 2y + 2z = 0$$
$$-6w + 12x + 3y - 4z = 0.$$

Ans. The equations can be written in the matrix form

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 4 & -6 & -2 & 2 \\ -6 & 12 & 3 & -4 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = 0 \Rightarrow AX = 0 \rightarrow (1)$$

$$\Rightarrow R_2/2 = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 2 & -3 & -1 & 1 \\ -6 & 12 & 3 & -4 \end{bmatrix}$$

$R_2 - R_1$.

$$R_3 + 3R_1 = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 0 & -6 & 0 & 2 \\ 0 & 21 & 0 & -7 \end{bmatrix}$$

$$\Rightarrow R_2/2,\ R_3/7 \Rightarrow \begin{bmatrix} 2 & 3 & -1 & -1 \\ 0 & -3 & 0 & 1 \\ 0 & 3 & 0 & -1 \end{bmatrix}$$

$R_3 \longrightarrow R_3 + R_2$

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 0 & -3 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow (2)$$

$\Rightarrow \rho(A) = 2 < 4(n)$

now, $n - r = 4 - 2 = 2$ [~~linearly~~ independently solutions]

Then the system is consistant ~~and~~ has an infinitely number of solutions.

from (2),

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 0 & -3 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$2w + 3x - y - z = 0$

$2w + 3x - y - z = 0$, $-3x + w = 0$

let $w = k_1$ $y = k_2$

$x = \frac{k_1}{3}$

$\Rightarrow 2w + \cancel{k_1} - k_2 - \cancel{k_1} = 0$

$\Rightarrow w = \frac{k_2}{2}$

Solve the system of equations

$2x - 3y + z = 0$

$4x + 9y + z = 0$

$8x - 27y + z = 0$

The given system of equations can be written in the matrix form

$$\Rightarrow \begin{bmatrix} 2 & -3 & 1 \\ 4 & 9 & 1 \\ 8 & -27 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow AX = 0 \rightarrow (1)$$

$$\Rightarrow A = \begin{bmatrix} 2 & -3 & 1 \\ 4 & 9 & 1 \\ 8 & -27 & 1 \end{bmatrix}$$

$\Rightarrow R_2 \rightarrow R_2 - 2R_1$
$R_3 \rightarrow R_3 - 4R_1$

$$= \begin{bmatrix} 2 & -3 & 1 \\ 0 & 15 & -1 \\ 0 & -15 & -3 \end{bmatrix}$$

$\Rightarrow R_3 \rightarrow R_3 + R_2$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 1 \\ 0 & 15 & -1 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\rho(A) = 3 = n(3)$

Here The system is consistance and has Trivial solution

$\Rightarrow x=0,\ y=0,\ z=0.$

3. Show the The only real number λ for which The system of equations

$x+2y+3z = \lambda x$
$3x+y+2z = \lambda y$
$2x+3y+z = \lambda z$

has a non-trivial solution. Which is 6. and also solve Them. when $\lambda = 6$.

The given system of equations can be written as

$(1-\lambda)x + 2y + 3z = 0$, $3x + (1-\lambda)y + 2z = 0$, $2x + 3y + (1-\lambda)z = 0$

Now the matrix notation

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

∵ The system has a non-trivial solution

$\Rightarrow |A| = 0$

$$\Rightarrow \begin{bmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{bmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

$R_1 \to R_1 + R_2 + R_3$

$$R_1 = \begin{vmatrix} 6-\lambda & 6-\lambda & 6-\lambda \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

$$= (6-\lambda) \begin{vmatrix} 1 & 1 & 1 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0.$$

$\Rightarrow$ $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$= (6-\lambda) \begin{vmatrix} 1 & 0 & 0 \\ 3 & -\lambda-2 & -1 \\ 2 & 1 & -\lambda-1 \end{vmatrix} = 0$$

$\Rightarrow 6-\lambda=0$, $\begin{vmatrix} 1 & 0 & 0 \\ 3 & -\lambda-2 & -1 \\ 2 & 1 & -\lambda-1 \end{vmatrix} = 0$

$\Rightarrow 6-\lambda=0$.

$\Rightarrow \lambda=6$

Substituting $\lambda=6$ in eq (1)

$$\Rightarrow \begin{bmatrix} -5 & 2 & 3 \\ 3 & -5 & 2 \\ 2 & 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_2 \to 5R_2 + 3R_1$

$R_3 \to 5R_3 + 2R_1$

$$\Rightarrow \begin{bmatrix} -5 & 2 & 3 \\ 0 & -19 & 19 \\ 0 & 19 & -19 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_2/19$, $R_3/19$ $\begin{bmatrix} -5 & 2 & 3 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$R_3 \to R_3 + R_2$

$$= \begin{bmatrix} -5 & 2 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\rho(A) = 2 < n(3)$

$n-r = 3-2 = 1$ [linearly independent solution]

$\Rightarrow -5x + 2y + 3z = 0$

$-y + z = 0$

let $z = k$

$y = k$

and $x = k$

4. Determine the value of b such that The system of homogeneouse equations

$2x + y + z = 0$

$x + y + 3z = 0$ has a non-trivial solution and find the solutions.

$4x + 3y + bz = 0$.

$\Rightarrow$ The given equation system of equations can be written in the matrix form

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 3 \\ 4 & 3 & b \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

given system has non-trivial solution then

$|A| = 0$

$$\Rightarrow \begin{vmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & b \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} \overset{+}{2} & \overset{-}{1} & \overset{+}{2} \\ 1 & 1 & 3 \\ 4 & 3 & b \end{vmatrix}$$

$\Rightarrow 2(b-9) - 1(b-12) + 2(3-4)$

$\Rightarrow 2b - 18 - b + 12 - 2 = 0$

$$b - 8 = 0$$
$$b = 8$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 4 & 3 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $R_2 \to 2R_2 - R_1$

$R_3 \to R_3 - 2R_1$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & 4 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$R_2 \to R_2 - R_3$

$$= \begin{bmatrix} 2 & 1 & 2 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 2 < n(3)$$

$\Rightarrow$ $n - r = 3 - 2 = 1$ linearly independent solution

$\Rightarrow$ $z = k$

$\Rightarrow$ $y + 4k = 0$

$y = -4k$

$2x - 4k + 2k = 0$

$2x - 2k = 0$

$x = k$

solve

$$x+3y+2z=0$$
$$2x-y+3z=0$$
$$3x-5y+4z=0$$
$$x+17y+4z=0$$

The given system of equations can be written in the matrix form

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$$

$\Rightarrow$ $R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - R_1$

$$= \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$$

$\Rightarrow R_4 \to R_4 + R_3$

$$= \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3/2$
$$= \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow R_3 \to R_3 - R_2$

$$= \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2 < n(3)$

$\therefore n - r = 3 - 2 = 1$ linearly independent solution

$\Rightarrow z = k$

$\Rightarrow -7y - k = 0$

$7y = -k$

$y = -1/7\, k$

$\Rightarrow x + 3\left(\frac{-1}{7}\right)k + 2(k) = 0$

$\Rightarrow x + \frac{-3}{7}k + 2k = 0$

$\Rightarrow \frac{7x - 3k + 14k}{7} = 0$

$\Rightarrow 7x + 11k = 0$

$x = \frac{-11}{7}k$

Solve.

$$2x_1 - 2x_2 + x_3 = \lambda x_1$$

$$2x_1 - 3x_2 + 2x_3 = \lambda x_2$$

$$-x_1 + 2x_2 = \lambda x_3$$ can poses a non thivial solutions if and only if $\lambda = 1$ or -3.