## Math Formula on Co-Ordinate

### ● Rectangular Cartesian Co-ordinates:

(i) If the pole and initial line of the polar system coincides respectively with the origin and positive x-axis of the Cartesian system and $(x, y)$, $(r, \theta)$ be the Cartesian and polar co-ordinates respectively of a point P on the plane then,

$x = r \cos \theta$, $y = r \sin \theta$ and $r = \sqrt{(x^2 + y^2)}$, $\theta = \tan^{-1}(y/x)$.

(ii) The distance between two given points $P (x_1, y_1)$ and $Q (x_2, y_2)$ is

$PQ = \sqrt{\{(x_2 - x_1)^2 + (y_2 - y_1)^2\}}$.

(iii) Let $P (x_1, y_1)$ and $Q (x_2, y_2)$ be two given points.

(a) If the point R divides the line-segment PQ internally in the ratio m : n, then the co-ordinates of R

are $\{(mx_2 + nx_1)/(m + n)\ ,\ (my_2 + ny_1)/(m + n)\}$.

(b) If the point R divides the line-segment PQ externally in the ratio m : n, then the co-ordinates of R

are $\{(mx_2 - nx_1)/(m - n),\ (my_2 - ny_1)/(m - n)\}$.

(c) If R is the mid-point of the line-segment PQ, then the co-ordinates of R are

$\{(x_1 + x_2)/2,\ (y_1 + y_2)/2\}$.

(iv) The co-ordinates of the centroid of the triangle formed by joining the points $(x_1, y_1)$ , $(x_2, y_2)$ and

$(x_3, y_3)$ are $(\{x_1 + x_2 + x_3\}/3\ ,\ \{y_1 + y_2 + y_3\}/3$

(v) The area of a triangle formed by joining the points $(x_1, y_1)$, $(x_2, y_2)$ and $(x_3, y_3)$ is

$\frac{1}{2}\ |\ y_1 (x_2 - x_3) + y_2 (x_3 - x_1) + y_3 (x_1 - x_2)\ |$ sq. units

or, $\frac{1}{2}\ |\ x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)\ |$ sq. units.

● **Straight Line:**

(i) The slope or gradient of a straight line is the trigonometric tangent of the angle $\theta$ which the line makes with the positive directive of x-axis.

(ii) The slope of x-axis or of a line parallel to x-axis is zero.

(iii) The slope of y-axis or of a line parallel to y-axis is undefined.

(iv) The slope of the line joining the points $(x_1, y_1)$ and $(x_2, y_2)$ is $m = (y_2 - y_1)/(x_2 - x_1)$.

(v) The equation of x-axis is $y = 0$ and the equation of a line parallel to x-axis is $y = b$.

(vi) The equation of y-axis is $x = 0$ and the equation of a line parallel to y-axis is $x = a$.

(vii) The equation of a straight line in

(a) slope-intercept form: $y = mx + c$ where m is the slope of the line and c is its y-intercept;

(b) point-slope form: $y - y_1 = m(x - x_1)$ where m is the slope of the line and $(x_1, y_1)$ is a given point on the line;

(c) symmetrical form: $(x - x_1)/\cos\theta = (y - y_1)/\sin\theta = r$, where $\theta$ is the inclination of the line, $(x_1, y_1)$ is a given point on the line and r is the distance between the points $(x, y)$ and $(x_1, y_1)$;

(d) two-point form: $(x - x_1)/(x_2 - x_1) = (y - y_1)/(y_2 - y_1)$ where $(x_1, y_1)$ and $(x_2, y_2)$ are two given points on the line;

(e) intercept form: $^x/_a + {}^y/_b = 1$ where a = x-intercept and b = y-intercept of the line;

(f) normal form: $x\cos\alpha + y\sin\alpha = p$ where p is the perpendicular distance of the line from the origin and $\alpha$ is the angle which the perpendicular line makes with the positive direction of the x-axis.

(g) general form: $ax + by + c = 0$ where a, b, c are constants and a, b are not both zero.

(viii) The equation of any straight line through the intersection of the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is $a_1x + b_1y + c + k(a_2x + b_2y + c_2) = 0$ $(k \neq 0)$.

(ix) If $p \neq 0$, $q \neq 0$, $r \neq 0$ are constants then the lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent if $P(a_1x + b_1y + c_1) + q(a_2x + b_2y + c_2) + r(a_3x + b_3y + c_3) = 0$.

(x) If $\theta$ be the angle between the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ then

$$\tan \theta = \pm (m_1 - m_2)/(1 + m_1 m_2);$$

(xi) The lines $y= m_1x + c_1$ and $y = m_2x + c_2$ are

(a) parallel to each other when $m_1 = m_2$;

(b) perpendicular to one another when $m_1 \cdot m_2 = - 1$.

(xii) The equation of any straight line which is

(a) parallel to the line $ax + by + c = 0$ is $ax + by = k$ where k is an arbitrary constant;

(b) perpendicular to the line $ax + by + c = 0$ is $bx - ay = k_1$ where $k_1$ is an arbitrary constant.

(xiii) The straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are identical if $a_1/a_2 = b_1/b_2 = c_1/c_2$.

(xiv) The points $(x_1, y_1)$ and $(x_2, y_2)$ lie on the same or opposite sides of the line $ax + by + c = 0$ according as $(ax_1 + by_1 + c)$ and $(ax_2 + by_2 + c)$ are of same sign or opposite signs.

(xv) Length of the perpendicular from the point (x1, y1) upon the line $ax + by + c = 0$ is

$$|(ax_1 + by_1 + c)|/\sqrt{(a^2 + b^2)}.$$

(xvi) The equations of the bisectors of the angles between the lines $a_1x + b_1y + c_1 = 0$ and

$a_2x + b_2y + c_2 =0$ are $(a_1x + b_1y + c_1)/\sqrt{(a_1^2 + b_1^2)} = \pm (a_2x + b_2y + c_2)/\sqrt{(a_2^2 + b_2^2)}$.

● **Circle:**

(i) The equation of the circle having centre at the origin and radius a units is $x^2 + y^2 = a^2 \ldots (1)$

The parametric equation of the circle (1) is $x = a \cos \theta$, $y = a \sin \theta$, $\theta$ being the parameter.

(ii) The equation of the circle having center at $(\alpha, \beta)$ and radius a units is $(x - \alpha)^2 + (y - \beta)^2 = a^2$.

(iii) The equation of the circle in general form is $x^2 + y^2 + 2gx + 2fy + c = 0$ The centre of this circle is at $(-g, -f)$ and radius $= \sqrt{(g^2 + f^2 - c)}$

(iv) The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a circle if $a = b\ (\neq 0)$ and $h = 0$.

(v) The equation of a circle concentric with the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $x^2 + y^2 + 2gx + 2fy + k = 0$ where k is an arbitrary constant.

(vi) The equation of the circle with the given points $(x_1, y_1)$ and $(x_2, y_2)$ as the ends of a diameter is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

● **Parabola:**

(i) Standard equation of parabola is $y^2 = 4ax$. Its vertex is the origin and axis is x-axis.

(ii) Other forms of the equations of parabola:

(a) $x^2 = 4ay$. Its vertex is the origin and axis is y-axis.

(b) $(y - \beta)^2 = 4a(x - \alpha)$. Its vertex is at $(\alpha, \beta)$ and axis is parallel to x-axis.

(c) $(x - \alpha)^2 = 4a(y - \beta)$. Its vertex is at $(a, \beta)$ and axis is parallel to y-axis.

(iii) $x = ay^2 + by + c$ $(a \neq o)$ represents equation of the parabola whose axis is parallel to x-axis.

(iv) $y = px^2 + qx + r$ $(p \neq o)$ represents equation of the parabola whose axis is parallel to y-axis.

(v) The parametric equations of the parabola $y^2 = 4ax$ are $x = at^2$ , $y = 2at$, t being the parameter.

(vi) The point $(x_1, y_1)$ lies outside, on or inside the parabola $y^2 = 4ax$ according as $y_1^2 = 4ax_1 >, =$ or,$<0$

● **Ellipse:**

(i) Standard equation of ellipse is $x^2/a^2 + y^2/b^2 = 1$ ……….(1)

(a) Its centre is the origin and major and minor axes are along x and y-axes respectively ; length of major axis = 2a and that of minor axis = 2b and eccentricity = $e = \sqrt{[1 - (b^2/a^2)]}$

(b) The point $(x_1, y_1)$ lies outside, on or inside the ellipse (1) according as $x_1^2/a^2 + y_1^2/b^2 - 1 >, =$ or $< 0$.

(c) The parametric equations of the ellipse (1) are $x = a \cos \theta$, $y = b \sin \theta$ where $\theta$ is the eccentric angle of the point P (x, y) on the ellipse (1) ; $(a \cos \theta, b \sin \theta)$ are called the parametric co-ordinates of P.

(ii) Other forms of the equations of ellipse:

(a) $x^2/a^2 + y^2/b^2 = 1$. Its centre is at the origin and the major and minor axes are along y and x-axes respectively.

(b) $[(x - \alpha)^2]/a^2 + [(y - \beta)^2]/b^2 = 1$.

The centre of this ellipse is at $(\alpha, \beta)$ and the major and minor ones are parallel to x-axis and y-axis respectively.

● **Hyperbola:**

(i) Standard equation of hyperbola is $x^2/a^2 - y^2/b^2 = 1$ . . . (1)

(a) Its centre is the origin and transverse and conjugate axes are along x and y-axes respectively ; its length of transverse axis = 2a and that of conjugate axis = 2b and eccentricity = $e = \sqrt{[1 + (b^2/a^2)]}$.

(b) If S and S' be the two foci and P (x, y) any point on it then SP = ex - a, S'P = ex + a and S'P - SP = 2a.

(c) The point $(x_1, y_1)$ lies outside, on or inside the hyperbola (1) according as $x_1^2/a^2 - y_1^2/b^2 = -1 <, =$ or, $> 0$.

(d) The parametric equation of the hyperbola (1 ) are $x = a \sec \theta$, $y = b \tan \theta$ and the parametric co-ordinates of any point P on (1) are $(a \sec \theta, b \tan \theta)$.

(e) The equation of auxiliary circle of the hyperbola (1) is $x^2 + y^2 = a^2$.

(ii) Other forms of the equations of hyperbola:

(a) $y^2/a^2 - x^2/b^2 = 1$.

Its centre is the origin and transverse and conjugate axes are along y and x-axes respectively.

(b) $[(x - \alpha)^2]/a^2 - [(y - \beta)^2]/b^2 = 1$. Its centre is at $(\alpha, \beta)$ and transverse and conjugate axes are parallel to x-axis and y-axis respectively.

(iii) Two hyperbolas
$x^2/a^2 - y^2/b^2 = 1$ ………..(2) and $y^2/b^2 - x^2/a^2 = 1$ …….. (3)

are conjugate to one another. If $e_1$ and $e_2$ be the eccentricities of the hyperbolas (2) and (3) respectively, then
$b^2 = a^2 (e_1^2 - 1)$ and $a^2 = b^2 (e_2^2 - 1)$.

(iv) The equation of rectangular hyperbola is $x^2 - y^2 = a^2$ ; its eccentricity = $\sqrt{2}$.